

A Quintessence Problem in Brans-Dicke Theory with Varying Speed of Light

Subenoy Chakraborty,^{*} N. C. Chakraborty, and Ujjal Debnath[†]

Department of Mathematics, Jadavpur University, Calcutta-32, India.

(Dated: February 7, 2008)

It is shown that minimally coupled scalar field in Brans-Dicke theory with varying speed of light can solve the quintessence problem and it is possible to have a non-decelerated expansion of the present universe with BD-theory for anisotropic models without any matter.

PACS numbers:

I. INTRODUCTION

Recently, the matter field can give rise to an accelerated expansion [1] for the universe stems from the observational data regarding the luminosity-redshift relation of type Ia supernovae [2,3] up to about $z \sim 1$. This matter field is called 'quintessence matter' (shortly Q-matter). This Q-matter can behave like a cosmological constant [4,5] by combining positive energy density and negative pressure. So there must be this Q-matter either neglected or unknown responsible for this accelerated universe. At the present epoch, a lot of works has been done to solve this quintessence problem and most popular candidates for Q-matter has so far been a scalar field having a potential which generates a sufficient negative pressure. Since in a variety of inflationary models scalar fields have been used in describing the transition from the quasi-exponential expansion of the early universe to a power-law expansion, it is natural to try to understand the present acceleration of the universe, which has an exponential behaviour too, by constructing models where the matter responsible for such behaviour is also represented by a scalar field [6]. However, now we deal with the opposite task, i.e., we would like to describe the transition from a universe filled with dust-like matter to an exponentially expansion universe and scalar fields are not the only possibility but there are (of course) alternatives. Different forms for the quintessence energy have been proposed. They include a cosmological constant (or, more generally a variable cosmological term) a scalar field [4,7] a frustrated network of non-abelian cosmic strings and a frustrated network of domain wall [8,9]. All these proposal assume the Q-matter behaves as a perfect fluid with a linear barotropic equation of state and so some effort has been invested in determining its adiabatic index at the present epoch [10,11]. Recently, Chimento et al [12] showed that a combination of dissipative effects such as a bulk viscous stress and a quintessence scalar field gives an accelerated expansion for an open universe ($k = -1$). Very recently, Banerjee et al [13] also have shown that it is possible to have an accelerated universe with BD-theory in Friedmann model without any matter.

The possibility that the speed of light c might vary has recently attracted considerable attention [14-20]. In a cosmological setting, the variations in c have been shown to solve the cosmological puzzles - the horizon, flatness and Lambda problems of big-bang cosmology. The variations of velocity of light can also solve the quasi-lambda problems. For power-law variations in the velocity of light with the cosmological scale factors, Barrow et al [16] have shown that flatness problem can be solved. The Machian VSL scenario in which $c = c_0 a^n$, introduced by Barrow [15] has significant advantages to the phase transition scenario in which the speed of light changes suddenly from c^- to c^+ , preferred by Albrecht and Magueijo [14]. For changing c , the geometry of the universe is not affected. We have allowed a changing c to do the job normally done by "superluminal expansion". The

^{*}Electronic address: subenoyc@yahoo.co.in

[†]Electronic address: ujjaldebnath@yahoo.com

basic assumption is that a variable c does not induce corrections to curvature in the cosmological frame and that Einstein's equations, relating curvature to stress energy are still valid. The rationale behind this postulate is that c changes in the local Lorentzian frames associated with cosmological expansion. The effect is a special relativistic effect, not a gravitational effect. Therefore curvature should not feel a changing c . In a cosmological setting the postulate proposed implies that Brans-Dicke field equations remains valid even when $\dot{c} \neq 0$. Magueijo et al [21] find the metrics and variations in c associated with the counterpart of black holes, the outside of star and stellar collapse. The variation upon the theme are VSL theories which explicitly break local Lorentz invariance, such as the one proposed by Albrecht and Magueijo [14] and for which black hole solutions remain exclusive.

This paper investigates the possibility of obtaining a non-decelerating expansion ($q \leq 0$) for the universe in BD theory with varying speed of light in anisotropic models of the universe.

II. FIELD EQUATIONS AND SOLUTIONS

We consider the line-element of anisotropic space-time model

$$ds^2 = -c^2 dt^2 + a^2 dx^2 + b^2 d\Omega_k^2 \quad (1)$$

where a, b are functions of time only and

$$d\Omega_k^2 = \begin{cases} dy^2 + dz^2, & \text{when } k = 0 \quad (\text{Bianchi I model}) \\ d\theta^2 + \sin^2\theta d\phi^2, & \text{when } k = +1 \quad (\text{Kantowski-Sachs model}) \\ d\theta^2 + \sinh^2\theta d\phi^2, & \text{when } k = -1 \quad (\text{Bianchi III model}) \end{cases}$$

Here k is the spatial curvature index, so that the above three types [22] models are Euclidean, closed and semi-closed respectively.

Now, the BD-field equations with varying speed of light are

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} = -\frac{8\pi}{(3+2\omega)\phi} \left[(2+\omega)\rho_f + 3(1+\omega)\frac{p_f}{c^2} \right] - \omega \left(\frac{\dot{\phi}}{\phi} \right)^2 - \frac{\ddot{\phi}}{\phi} \quad (2)$$

$$\left(\frac{\dot{b}}{b} \right)^2 + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} = \frac{8\pi\rho_f}{\phi} - \frac{kc^2}{b^2} - \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \frac{\dot{\phi}}{\phi} + \frac{\omega}{2} \left(\frac{\dot{\phi}}{\phi} \right)^2 \quad (3)$$

and the wave equation for the BD scalar field ϕ is

$$\ddot{\phi} + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \dot{\phi} = \frac{8\pi}{3+2\omega} \left(\rho_f - \frac{3p_f}{c^2} \right) \quad (4)$$

Here the velocity of light c is an arbitrary function of time, ω is the BD coupling parameter. ρ_f and p_f are density and hydrostatic pressure respectively of the fluid distribution with barotropic equation of state

$$p_f = (\gamma_f - 1)\rho_f$$

(γ_f being the constant adiabatic index of the fluid with $0 \leq \gamma_f \leq 2$).

From the above field equations, we have the 'non-conservation' equation

$$\dot{\rho}_f + \left(\frac{\dot{a}}{a} + 2\frac{\dot{b}}{b} \right) \left(\rho_f + \frac{p_f}{c^2} \right) = \frac{k c \dot{c}}{4\pi b^2} \phi \quad (5)$$

As at the present epoch the universe is filled with cold matter (dust) with negligible pressure, so using $p_f = 0$, the equation (4) and (5) gives

$$\frac{d}{dt}(ab^2\dot{\phi}) = \frac{8\pi}{3+2\omega} ab^2 \rho_f \quad (6)$$

and

$$\frac{d}{dt}(ab^2\rho_f) = \frac{k a \phi}{8\pi} \frac{d}{dt}(c^2) \quad (7)$$

Now eliminating ρ_f between (6) and (7), we have

$$\frac{d^2}{dt^2}(ab^2\dot{\phi}) = \frac{k}{3+2\omega} a \phi \frac{d}{dt}(c^2) \quad (8)$$

Since the scale factors, scalar field and velocity of light are time dependent, so we shall consider the following cases to obtain exact analytic form for the variables assuming some of them in polynomial form or in exponential form.

Case I : The power law form of scale factors a, b and BD scalar field ϕ are assumed as

$$a(t) = a_0 t^\alpha, b(t) = b_0 t^\beta, \phi(t) = \phi_0 t^\mu \quad (9)$$

where a_0, b_0, ϕ_0 are positive constants and α, β, μ are real constants with the restriction $\alpha + 2\beta \geq 3$ (for accelerating universe).

If we use the relations (9) in (8), we have

$$c = c_0 t^{\beta-1} \quad (10)$$

where the positive constant c_0 is restricted as

$$b_0^2 \mu (\alpha + 2\beta + \mu - 1) (\alpha + 2\beta + \mu - 2) = \frac{2k c_0^2 (\beta - 1)}{3 + 2\omega} \quad (11)$$

Also from (6) the expression for ρ_f is

$$\rho_f = \rho_0 t^{\mu-2} \quad (12)$$

with

$$\rho_0 = \frac{3+2\omega}{8\pi} \phi_0 \mu (\alpha + 2\beta + \mu - 1)$$

For consistency of the field equations the restriction between the parameters is

$$\left(\alpha + 2\beta + \mu - \frac{1}{2} \right)^2 = \frac{\mu(\alpha + 2\beta + \mu - 1)}{\beta - 1} [(2 + \omega)(1 - \beta) - (\alpha - \mu)(3 + 2\omega)] + \frac{1}{4} \quad (13)$$

Case II : The exponential form of scale factors and BD scalar field are assumed as

$$a(t) = a_0 e^{\alpha t}, b(t) = b_0 e^{\beta t}, \phi(t) = \phi_0 e^{\mu t} \quad (14)$$

where a_0, b_0, ϕ_0 are positive constants and α, β, μ are real constants.

From (8), we have

$$c = c_0 e^{\beta t} \quad (15)$$

where the positive constant c_0 will satisfy

$$(3 + 2\omega)\mu(\alpha + 2\beta + \mu)^2 = \frac{4k\beta c_0^2}{b_0^2} \quad (16)$$

From (6), the expression for ρ_f is

$$\rho_f = \rho_0 e^{\mu t} \quad (17)$$

with $\rho_0 = \frac{(3+2\omega)}{8\pi} \phi_0 \mu (\alpha + 2\beta + \mu)$.

From field equations the relation between the parameters becomes

$$\text{either } \alpha + 2\beta + \mu = 0 \quad (18)$$

$$\text{or } (\alpha + 2\beta + \mu) \left\{ 1 + \frac{(3 + 2\omega)\mu}{\beta^2} \right\} = \mu(4 + 3\omega) \quad (19)$$

In both the cases we have the deceleration parameter

$$q = -1 \leq 0$$

Case III : In this case, we have assumed the power law form of scale factor b , velocity of light c and BD scalar field ϕ to be

$$b(t) = b_0 t^\beta, c(t) = c_0 t^\delta, \phi(t) = \phi_0 t^\mu \quad (20)$$

Using (20), we have from (8),

$$\frac{d^2}{dt^2} (at^{2\beta+\mu-1}) = \frac{2\delta k c_0^2}{(3 + 2\omega)\mu b_0^2} (at^{\mu+2\delta-1}) \quad (21)$$

To solve the differential equation we assume $\delta = \beta$. It is to be noted that the above differential equation can be solved for all k . But for $k = \pm 1$ the solutions are not consistent to the field equations. So we consider only $k = 0$. In this case the explicit solution is

$$\left. \begin{aligned} a(t) &= a_0 t^{-\frac{\omega+2}{3\omega+4}} + a_1 t^{\frac{2(\omega+1)}{3\omega+4}} \\ b(t) &= b_0 t^{\frac{2(\omega+1)}{3\omega+4}} \\ c(t) &= c_0 t^{\frac{2(\omega+1)}{3\omega+4}} \\ \phi(t) &= \phi_0 t^{\frac{2}{3\omega+4}} \\ \rho_f &= \frac{1}{4\pi} \frac{2\omega+3}{3\omega+4} \frac{\phi_0 a_1}{(a_0 + a_1 t)} t^{-\frac{3\omega+2}{3\omega+4}} \end{aligned} \right\} \quad (22)$$

with a_0, a_1 as integration constants.

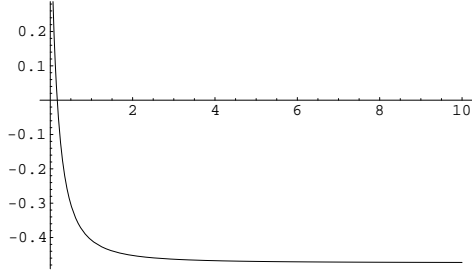


Fig.1

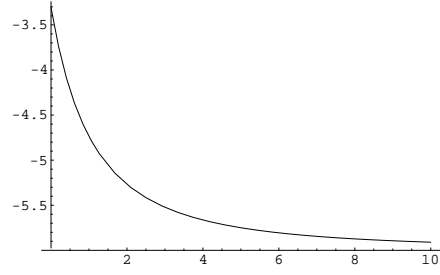


Fig.2

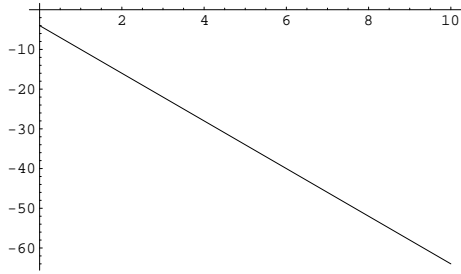


Fig.3

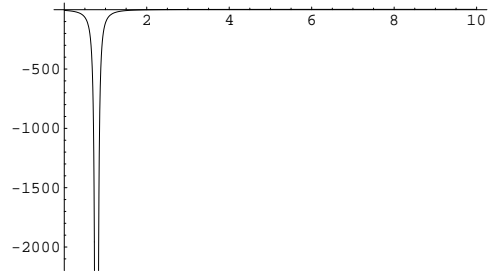


Fig.4

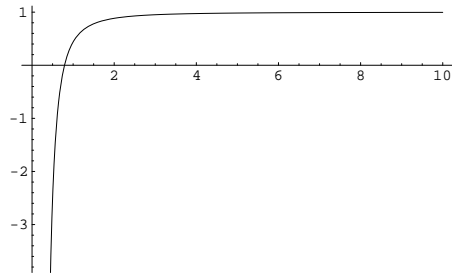


Fig.5

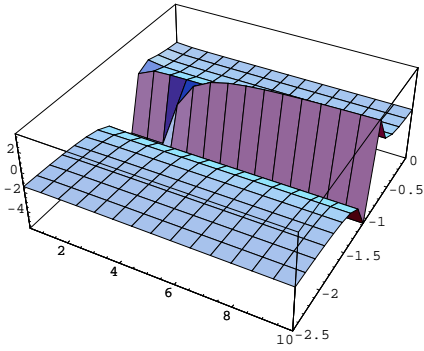


Fig.6

In Figs.1-5, we have shown the variations of q over t for various values of ω and the values of parameters $a_0 = a_1 = 1$. We have taken $\omega = -1.8, -1.5, -1, -0.87, -0.75$ respectively in Figs.1-5. In Fig.6, we have shown the variations of q over t and ω in the range $0 \leq t \leq 10$ and $-2.5 \leq \omega \leq 0$.

The deceleration parameter has the expression

$$q = -1 + \frac{3[(3\omega+2)(a_0+a_1 t)^2 + a_1^2 (3\omega+4)t^2]}{[(3\omega+2)(a_0+a_1 t) + a_1 (3\omega+4)t]^2},$$

which is finite for all t and ω except $t = -\frac{a_0(3\omega+2)}{6a_1(\omega+1)}$ and we have the non-decelerated universe for $-2 \leq \omega \leq -2/3$. Also we have the non-decelerated universe asymptotically except for $-1 \leq \omega \leq -1/2$. The variation of q over time has been shown graphically for different values of ω in figures 1 - 5.

III. CONFORMAL TRANSFORMATION : FLATNESS PROBLEM

One important aspect of this model is that it can solve the flatness problem as well. To see this we make a conformal transformation [23] as

$$\bar{g}_{\mu\nu} = \phi g_{\mu\nu} \quad (23)$$

which enables us to identify the energy contributions from different components of matter very clearly.

In this section, we have developed the BD theory in Jordan frame and to introduce the Einstein frame, we make the following transformations :

$$d\eta = \sqrt{\phi} a, \bar{a} = \sqrt{\phi} a, \bar{b} = \sqrt{\phi} b, \psi = \ln \phi, \bar{\rho}_f = \phi^{-2} \rho_f, \bar{\rho}_\psi = \phi^{-2} \rho_\psi, \bar{p}_f = \phi^{-2} p_f, \bar{p}_\psi = \phi^{-2} p_\psi \quad (24)$$

So the field equations (2) - (4) transformed to

$$\frac{\bar{a}''}{\bar{a}} + 2\frac{\bar{b}''}{\bar{b}} = -4\pi \left(\bar{\rho}_f + \frac{3\bar{p}_f}{c^2} \right) - \frac{(3+2\omega)}{2} \psi'^2 \quad (25)$$

$$\left(\frac{\bar{b}'}{\bar{b}} \right)^2 + 2\frac{\bar{a}'}{\bar{a}} \frac{\bar{b}'}{\bar{b}} + \frac{kc^2}{\bar{b}^2} = 8\pi \bar{\rho}_f + \frac{(3+2\omega)}{4} \psi'^2 \quad (26)$$

and

$$\psi'' + \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}} \right) \psi' = \frac{8\pi}{3+2\omega} \left(\bar{\rho}_f - \frac{3\bar{p}_f}{c^2} \right) \quad (27)$$

where $' \equiv \frac{d}{d\eta}$.

The scalar field ψ (massless) behaves like a ‘stiff’ perfect fluid with equation of state

$$\bar{p}_\psi = \bar{\rho}_\psi = \frac{\psi'^2}{16\pi G} \quad (28)$$

If the velocity of light is constant, then in Einstein frame total stress-energy tensor is conserved but there is an exchange of energy between the scalar field and normal matter according to the following equation

$$\bar{\rho}_f + \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}} \right) \left(\bar{\rho}_f + \frac{\bar{p}_f}{c^2} \right) = - \left[\bar{\rho}_\psi + \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}} \right) (\bar{\rho}_\psi + \bar{p}_\psi) \right] = -\frac{\psi'}{2} \left(\bar{\rho}_f - \frac{3\bar{p}_f}{c^2} \right) \quad (29)$$

On the other hand, if the velocity of light varies then we have separate ‘non-conservation’ equations

$$\bar{\rho}_f + \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}} \right) \left(\bar{\rho}_f + \frac{\bar{p}_f}{c^2} \right) = -\frac{\psi'}{2} \left(\bar{\rho}_f - \frac{3\bar{p}_f}{c^2} \right) + \frac{kc c'}{4\pi G \bar{b}^2} \quad (30)$$

and

$$\bar{\rho}_\psi + \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}} \right) (\bar{\rho}_\psi + \bar{p}_\psi) = \frac{\psi'}{2} \left(\bar{\rho}_\psi - \frac{3\bar{p}_\psi}{c^2} \right) \quad (31)$$

Thus combining the two energy densities, we have from the above equations, the equation for the conservation for the total energy is

$$\bar{\rho}' + 3\gamma\bar{H}\bar{\rho} = 0 \quad (32)$$

Here $\bar{H} = \frac{1}{3} \left(\frac{\bar{a}'}{\bar{a}} + 2\frac{\bar{b}'}{\bar{b}} \right)$ is the Hubble parameter in the Einstein frame and γ is the net barotropic index defined as

$$\gamma\bar{\Omega} = \gamma_f\bar{\Omega}_f + \gamma_\psi\bar{\Omega}_\psi \quad (33)$$

where

$$\bar{\Omega} = \bar{\Omega}_f + \bar{\Omega}_\psi = \frac{\bar{\rho}}{3\bar{H}^2} \quad (34)$$

is the dimensionless density parameter.

From equations (26) and (32), we have the evolution equation for the density parameter as

$$\bar{\Omega}' = \bar{\Omega}(\bar{\Omega} - 1)[\gamma\bar{H}_a + 2(\gamma - 1)\bar{H}_b] \quad (35)$$

where $\bar{H}_a = \frac{\bar{a}'}{\bar{a}}$ and $\bar{H}_b = \frac{\bar{b}'}{\bar{b}}$.

This equation in $\bar{\Omega}$ shows that $\bar{\Omega} = 1$ is a possible solution of it and for stability of this solution, we have

$$\gamma < \frac{2}{3} \quad (36)$$

Since the adiabatic indices do not change due to conformal transformation so we take $\gamma_f = 1$ (since $p_f = 0$) and $\gamma_\psi = 2$. Hence from (33) and (34), we have

$$\gamma = \frac{\bar{\Omega}_f + 2\bar{\Omega}_\psi}{\bar{\Omega}_f + \bar{\Omega}_\psi} \quad (37)$$

Now due to upper limit of γ , we must have the inequality

$$\bar{\Omega}_f < 4|\bar{\Omega}_\psi| \quad (38)$$

according as γ is restricted by (36).

From the field equation (26), the curvature parameter $\bar{\Omega}_k = -kc^2/\bar{b}^2$ vanishes for the solution $\bar{\Omega} = 1$. So for BD-scalar field it is possible to have a stable solution corresponding to $\bar{\Omega} = 1$ and hence the flatness problem can be solved.

IV. CONCLUDING REMARKS

We have performed an extensive analysis of solutions to Brans-Dicke theories with varying speed of light. For the power-law forms and exponential forms of the cosmological scale factors and scalar field in *case I* and *II* respectively, we have the velocity of light in the same form and we identified the cases where the quintessence problem can be solved for some restrictions on parameters. In the *case I*, we always have the accelerated universe.

For figures 1 and 2, q decreases to a fixed negative value asymptotically for $\omega = -1.8$ and $\omega = -1.5$. For figure 3, q is linear with time and always negative for $\omega = -1$. For figure 4, q decreases and then there is a singularity and after that q increases. In figure 5, q increases to a fixed positive value asymptotically. From figure 6, we conclude that the decelerated parameter is negative whenever $-2 < \omega < -2/3$. In this range q decreases till $t = -\frac{a_0(3\omega+2)}{6a_1(\omega+1)}$ and then increases to a fixed positive value asymptotically.

Along with providing a non-decelerating solution, it can solve the flatness problem also. In fact, it has been shown that $\bar{\Omega} = 1$ could be a stable solution in this model.

Acknowledgement:

The authors are thankful to the Relativity and Cosmology Research Centre, Department of Physics, Jadavpur University for helpful discussion. One of the authors (U.D) is thankful to CSIR (Govt. of India) for awarding a Junior Research Fellowship.

References:

- [1] N. A. Bachall, J. P. Ostriker, S. Perlmutter, P. J. Steinhardt, *Science* **284** (1999) 1481.
- [2] Perlmutter S et al 1998 *Nature* (London) **391** 51; 1999 *Astrophys J* **517** 565.
- [3] Riess A G et al 1998 *Astrophys J* **116** 1009; Garnavich P M et al 1998 *Astrophys J* **509** 74.
- [4] R. R. Caldwell, R. Dave and P. J. Steinhardt, *Phys. Rev. Lett.* **80**, 1582 (1998) [*astro-ph* / 9708069].
- [5] V. Sahni, A. A. Starobinsky, *Int. J. Mod. Phys. D* **9** 373 (2000) [*astro-ph*/9904398].
- [6] A. A. Starobinsky, *JEPT Lett.* **68** (1998) 757; T. D. Saini, S. Roychoudhury, V. Sahni, A. A. Starobinsky, *Phys. Rev. Lett.* **85** (2000) 1162.
- [7] Faraoni V 2000 *Phys. Rev. D* **62** 023504.
- [8] Bucher M and Spergel D 1999 *Phys. Rev. D* **60** 043505.
- [9] Battye R A, Bucher M and Spergel D "Domain wall dominated universe" ,*astro-ph*/990847.
- [10] L. Wang, R. Caldwell, J. P. Ostriker and P. J. Steinhardt, *Astrophys. J.* **530** 17 (2000).
- [11] I. Waga and P. M. R. Miceli, *Phys. Rev. D* **59** 103507 (1999).
- [12] Chimento L P, Jakubi A S and Pavo'n D 2000, *Phys. Rev. D* **62** 063508 (Preprint *astro-ph*/005070). [13] Banerjee N and Pavo'n D 2001 *Phys. Rev. D* **63** 043504.
- [14] A. Albrecht and J. Magueijo, *Phys. Rev. D* **59** 043516 (1999).
- [15] J. D. Barrow, *Phys. Rev. D* **59** 043515 (1999).
- [16] J. D. Barrow and J. Magueijo, *Phys. Lett. B* **443** 104 (1998); *Phys. Lett. B* **447** 246 (1999); *Class. Quantum Grav.* **16** 1435 (1999).
- [17] J. D. Barrow and J. Magueijo, *Astrophys. J. Lett.* **532** L87 (2000); J. Magueijo, *Phys. Rev. D* **62** 103521 (2000).
- [18] J. Moffat, *Int. J. Mod. Phys. D* **2** 351 (1993); *Fund. Phys.* **23** 411 (1993).
- [19] M. A. Clayton and J. W. Moffat, *Phys. Lett. B* **460** 263 (1999); *Phys. Lett. B* **477** 269 (2000).
- [20] E. Kiritsis, *J. High Energy Phys.* **10** 010 (1999).
- [21] J. Magueijo, *Phys. Rev. D* **63** 043502 (2001).
- [22] Thorne K S 1967 *Astrophys. J.* **148** 51.
- [23] Faraoni V, Gunzig E and Nardone P 1999, *Fundam. Cosm. Phys.* **20** 121.